Chapter 8 - Potential energy and conservation of energy

David J. Starling
Penn State Hazleton
PHYS 211
Work $W$ is how energy is transferred to or from a system.
Forces can be split into two categories known as **conservative and non-conservative**.
Conservative vs. Non-conservative Forces

*Forces can be split into two categories known as conservative and non-conservative.*

In one case, energy is “lost.”
Conservative vs. Non-conservative Forces

Consider:

- Two or more objects (e.g., earth + box)
Conservative vs. Non-conservative Forces

Consider:

- Two or more objects (e.g., earth + box)
- A force between them (e.g., \( mg \))
Conservative vs. Non-conservative Forces

Consider:

- Two or more objects (e.g., earth + box)
- A force between them (e.g., \(mg\))
- One object moves and work \(W_1\) is done (lift box up)
Consider:

- Two or more objects (e.g., earth + box)
- A force between them (e.g., \( mg \))
- One object moves and work \( W_1 \) is done (lift box up)
- The object returns and work is done \( W_2 \) (set box down)
Conservative vs. Non-conservative Forces

Consider:

- Two or more objects (e.g., earth + box)
- A force between them (e.g., \(mg\))
- One object moves and work \(W_1\) is done (lift box up)
- The object returns and work is done \(W_2\) (set box down)

If \(W_1 = -W_2\) is always true, no net work was done and that force is conservative.
The net work done by a conservative force on a particle moving around any closed path is zero.
Conservative vs. Non-conservative Forces

The net work done by a conservative force on a particle moving around any closed path is zero.

Equivalently: The net work done by a conservative force on a particle moving from point \( a \) to point \( b \) is independent of the path.
Conservative: Gravity, Spring, Electric Force
Conservative vs. Non-conservative Forces

**Conservative:** Gravity, Spring, Electric Force

**Non-conservative:** Friction, Air Drag
Lecture Question 8.1
A mountain climber pulls a supply pack up the side of a mountain at constant speed. Which one of the following statements concerning this situation is false?

(a) The net work done by all the forces acting on the pack is zero joules.
(b) The work done on the pack by the normal force of the mountain is zero joules.
(c) The work done on the pack by gravity is zero joules.
(d) The gravitational potential energy of the pack is increasing.
(e) The climber does "positive" work in pulling the pack up the mountain.
Potential Energy $U$ is a form of energy associated with a conservative force between a system of objects.
Potential Energy $U$ is a form of energy associated with a conservative force between a system of objects.

The spring force is conservative; it stores potential energy and then releases it.
Potential Energy $U$ is a form of energy associated with a conservative force between a system of objects.
Potential Energy $U$ is a form of energy associated with a conservative force between a system of objects.

The gravitational force is conservative; it stores potential energy and then releases it when the object is dropped.
For a conservative force, the change in Potential Energy \( \Delta U \) is defined as minus the work done by that conservative force.

\[
\Delta U = -W = - \int_{x_i}^{x_f} F(x)dx
\]
For a conservative force, the change in Potential Energy $\Delta U$ is defined as minus the work done by that conservative force.

$$\Delta U = -W = - \int_{x_i}^{x_f} F(x) \, dx$$

Example: If you lift an object, gravity does negative work, so $\Delta U > 0$. 
The change in gravitational potential energy of an object near Earth’s surface is

\[ \Delta U = mg(\Delta y). \]
The change in gravitational potential energy of an object near Earth’s surface is

\[ \Delta U = mg(\Delta y). \]

Note: only the change is important!
The change in spring potential energy is

$$\Delta U = \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2.$$
The change in spring potential energy is

$$\Delta U = \frac{1}{2} k x_f^2 - \frac{1}{2} k x_i^2.$$  

Note again: only the change is important!
Potential energy and conservative forces are related through a derivative/integral (by definition).

\[ \Delta U = -W = - \int F(x) dx \approx -F(x) \Delta x \]
Potential energy and conservative forces are related through a derivative/integral (by definition).

\[ \Delta U = -W = - \int F(x)dx \approx -F(x)\Delta x \]

\[ F(x) = -\frac{\Delta U}{\Delta x} \rightarrow F(x) = -\frac{dU}{dx} \]
Potential energy and conservative forces are related through a derivative/integral (by definition).

\[ \Delta U = -W = - \int F(x)\,dx \approx -F(x)\Delta x \]

\[ F(x) = -\frac{\Delta U}{\Delta x} \rightarrow F(x) = -\frac{dU}{dx} \]
Potential energy and conservative forces are related through a derivative/integral (by definition).

\[ \Delta U = -W = - \int F(x) \, dx \approx -F(x) \Delta x \]

\[ F(x) = -\frac{\Delta U}{\Delta x} \rightarrow F(x) = -\frac{dU}{dx} \]
Conservation of Energy

*The mechanical energy of a system is the sum of its kinetic and potential energies.*

\[
E_{mec} = K + U
\]

\[
\Delta E_{mec} = \Delta K + \Delta U
\]
Conservation of Energy

The mechanical energy of a system is the sum of its kinetic and potential energies.

\[ E_{mec} = K + U \]
\[ \Delta E_{mec} = \Delta K + \Delta U \]

If a system has only conservative forces:

\[ \Delta U = -W \]
Conservation of Energy

The mechanical energy of a system is the sum of its kinetic and potential energies.

\[ E_{mec} = K + U \]
\[ \Delta E_{mec} = \Delta K + \Delta U \]

If a system has only conservative forces:

\[ \Delta U = -W \]
\[ \Delta K = W \text{ (last chapter)} \]
The mechanical energy of a system is the sum of its kinetic and potential energies.

\[ E_{mec} = K + U \]
\[ \Delta E_{mec} = \Delta K + \Delta U \]

If a system has only conservative forces:

\[ \Delta U = -W \]
\[ \Delta K = W \text{ (last chapter)} \]
\[ \Delta U = -\Delta K \]
\[ \Delta K + \Delta U = 0 \]
\[ \Delta E_{mec} = 0 \]
The **mechanical energy** of a system is the sum of its kinetic and potential energies.

\[
E_{mec} = K + U
\]

\[
\Delta E_{mec} = \Delta K + \Delta U
\]

If a system has *only conservative forces*:

\[
\Delta U = -W
\]

\[
\Delta K = W \text{ (last chapter)}
\]

\[
\Delta U = -\Delta K
\]

\[
\Delta K + \Delta U = 0
\]

\[
\Delta E_{mec} = 0
\]

Mechanical energy is conserved but can transform from one type \((K\) or \(U\)) to another.
A pendulum is a good example of conservation of mechanical energy.
A roller coaster car travels down a hill and is moving at 18 m/s as it passes through a section of straight, horizontal track. The car then travels up another hill that has a maximum height of 15 m. If frictional effects are ignored, determine whether the car can reach the top of the hill. If it does reach the top, what is the speed of the car at the top?

(a) No, the car doesn’t make it up the hill.
(b) Yes, the car just barely makes it to the top and stops.
(c) Yes, and the car is moving at 5.4 m/s at the top.
(d) Yes, and the car is moving at 9.0 m/s at the top.
(e) Yes, and the car is moving at 18 m/s at the top.
An object subjected to a conservative force may have the following potential energy curve.
What happens if we place an object at rest at each of the 5 points shown?
Conservation of Energy

▶ $x_1$: falls to the right (unstable)
▶ $x_2$: sits in place and is restored if displaced (stable equilibrium)
▶ $x_3$: sits in place and is falls if displaced (unstable equilibrium)
▶ $x_4$: stable equilibrium
▶ $x_5$: unstable equilibrium
Conservation of Energy

The total energy in a system includes mechanical energy, thermal energy and internal energy.

\[ E = K + U + E_{th} + E_{int} \]
The total energy in a system includes mechanical energy, thermal energy and internal energy.

\[ E = K + U + E_{th} + E_{int} \]

The total energy in an isolated system is conserved:

\[ \Delta E = \Delta K + \Delta U + \Delta E_{th} + \Delta E_{int} = 0. \]
Internal energy can be transferred to kinetic energy, which can then be transferred to thermal energy.
Lecture Question 8.3
A 65 kg hiker eats a 250 calorie granola bar. Assuming the body converts this snack with an efficiency of 25%, what change of altitude could this hiker achieve by hiking up the side of a mountain before completely using the energy in the snack? (one food calorie is equal to 4186 joules)

(a) 270 m  
(b) 410 m  
(c) 650 m  
(d) 880 m  
(e) 1600 m
An external force can supply energy to a system.

\[ W = \Delta K + \Delta U = \Delta E_{\text{mec}} \]
An external force can supply energy to a system.

The lifting force supplies energy:

\[ W_{\text{lift}} = \Delta K + \Delta U = \Delta E_{\text{mech}} \]
An external force can supply energy to a system.

The lifting force supplies energy:

\[ W_{\text{lift}} = \Delta K + \Delta U = \Delta E_{\text{mech}} \]

(postive work!)
An external force can supply energy to a system.
An external force can supply energy to a system.

The pushing force supplies energy and friction sucks it away:

\[ W_{\text{push}} + W_{\text{friction}} = \Delta K + \Delta U = \Delta E_{\text{mech}} \]