Chapter 2 - Vectors

Roger: We have clearance, Clarence.

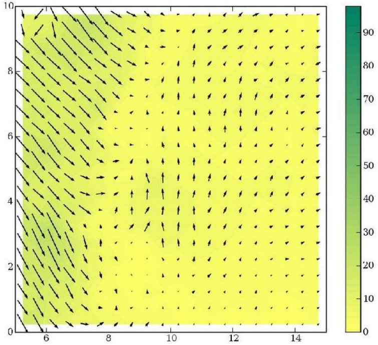
Clarence: Roger, Roger. What's our vector, Victor?

- from *Airplane!* (1980)

David J. Starling
Penn State Hazleton
PHYS 211

Vectors vs. Scalars

A **vector** *is a quantity that indicates both magnitude and direction.*



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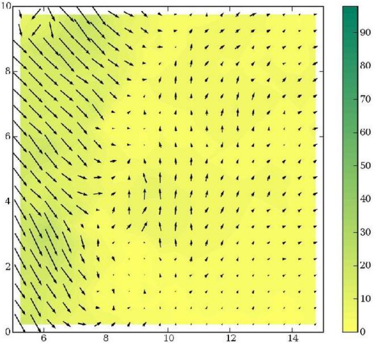
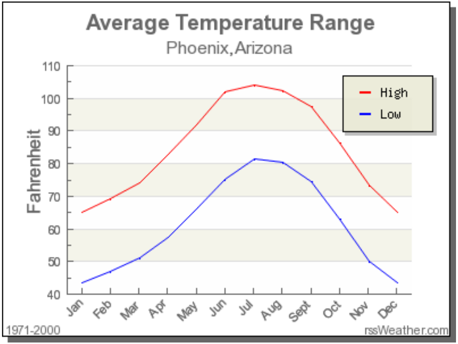


Examples: position, velocity and acceleration

Vectors vs. Scalars

A **scalar** *is a quantity that indicates only magnitude.*



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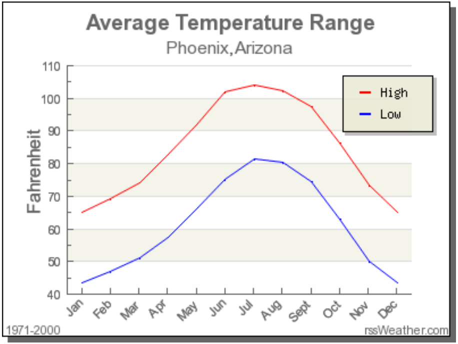
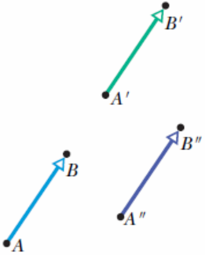


Examples: time, speed, temperature, distance

Vectors vs. Scalars

We represent a vector as an arrow with a direction and a length (magnitude).



(a)

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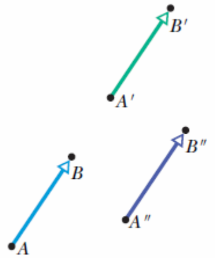
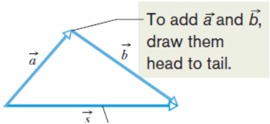


(a)

These are *displacement* vectors. Vectors are written mathematically as: $\vec{V}$ or $\mathbf{V}$.

Adding Vectors Geometrically

Vectors $\vec{a}$ and $\vec{b}$ can be added geometrically to give the sum $\vec{s} = \vec{a} + \vec{b}$.



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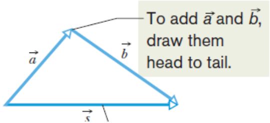

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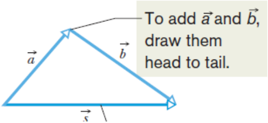


- This works for *any* vector (position, velocity, electric field, etc.)
- The size of the vector is called its **magnitude**

$$|\vec{s}| = s$$

Adding Vectors Geometrically

Here are some (familiar!) properties of vector addition.

(a) Commutative: $\vec{a} + \vec{b} = \vec{b} + \vec{a}$

(b) Associative: $(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$

(c) Additive inverse exists: $\vec{a} + (-\vec{a}) = \vec{0}$

(d) Subtraction: $\vec{a} - \vec{b} = \vec{a} + (-\vec{b}) = \vec{c}$

(e) Distributive: $m \times (\vec{a} + \vec{b}) = m\vec{a} + m\vec{b}$

Adding Vectors Geometrically

Lecture Question 2.1

You are standing in a soccer field. If you walk 10 m north, and then 3 m east, you arrive at point B. However, if you had walked 3 m east, and *then* 10 m north, you'd still arrive at point B. Which vector property does this demonstrate?

(a) Commutative

(b) Associative

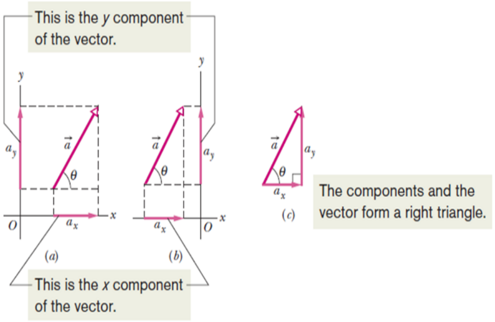
(c) Additive inverse

(d) Subtraction

(e) Distributive

Adding Vectors by Components

The component of a vector is the projection of the vector onto that axis.



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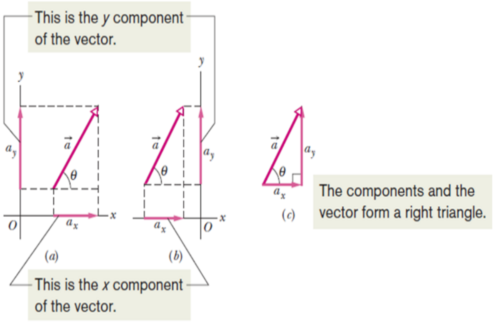
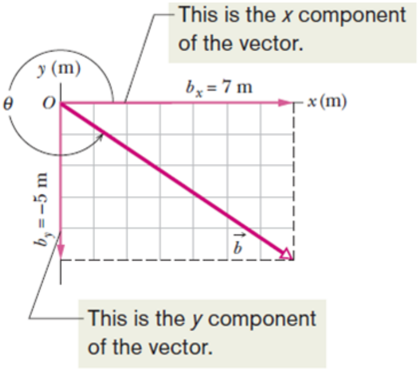


The projections are found using trigonometry

$$\sin(\theta) = a_y/a \text{ and } \cos(\theta) = a_x/a$$

Adding Vectors by Components

The same equations apply, even if the vector points in a different quadrant.



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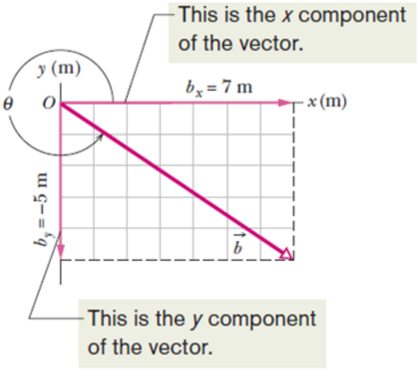
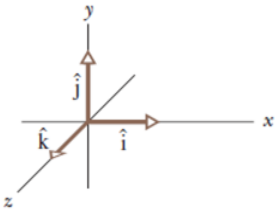


$$b = \sqrt{b_x^2 + b_y^2} \quad \text{and} \quad \tan(\theta) = \frac{b_y}{b_x}$$

Adding Vectors by Components

All **unit vectors** *have length/magnitude equal to one. They are written with a hat:* $\hat{a}$.



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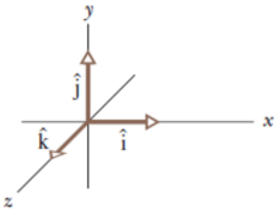
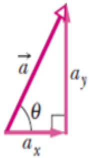


The common unit vectors are: $\hat{i}$, $\hat{j}$ and $\hat{k}$ pointing in the x, y and z directions.

Adding Vectors by Components

Vectors can be written in terms of their components and unit vectors.



$$\vec{a} = a_x\hat{i} + a_y\hat{j}$$

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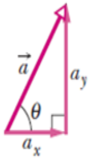

$$\vec{a} = a_x\hat{i} + a_y\hat{j}$$

a_x scales the $\hat{i}$ vector so that it has length $a_x \times 1$.

a_y scales the $\hat{j}$ vector so that it has length $a_y \times 1$.

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Adding two vectors can be done component by component.

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s_x &= a_x + b_x \\
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s_z &= a_z + b_z \\
\vec{s} &= s_x\hat{i} + s_y\hat{j} + s_z\hat{k} \\
&= (a_x + b_x)\hat{i} + (a_y + b_y)\hat{j} + (a_z + b_z)\hat{k}
\end{aligned}$$

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Subtracting two vectors is just like adding the negative of a vector.

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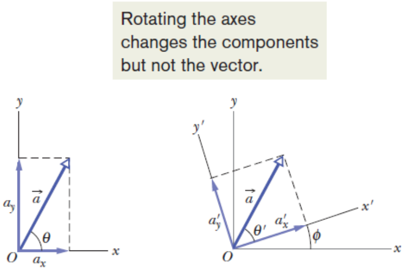
\end{aligned}$$

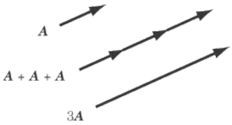
Adding Vectors by Components

The choice of axes is arbitrary. For any problem, you can choose the axes as you see fit.



Multiplying Vectors

1. A vector can be scaled (made larger or smaller) by a scalar:



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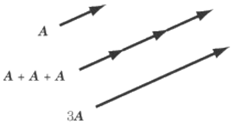

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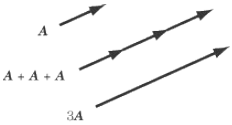

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- If $0 < c < 1$, $\vec{B}$ is shorter than $\vec{A}$

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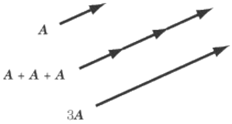

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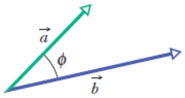
- If $c > 1$, $\vec{B}$ is c times larger than $\vec{A}$
- If $0 < c < 1$, $\vec{B}$ is shorter than $\vec{A}$
- If $c < 0$, $\vec{A}$ and $\vec{B}$ are in opposite directions.

Multiplying Vectors

2. The **scalar product** *is the multiplication of two vectors that results in a scalar.*



(a)

$$\vec{a} \cdot \vec{b} = ab\cos(\phi)$$

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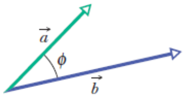


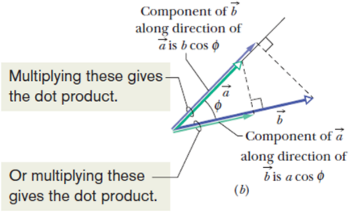
(*a*)

$$\vec{a} \cdot \vec{b} = ab\cos(\phi)$$

Note: $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$ (commutative)

Multiplying Vectors

The scalar product tells us how much two vectors are parallel.



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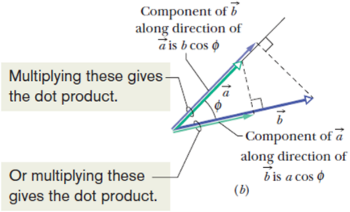

- By components: $\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z$

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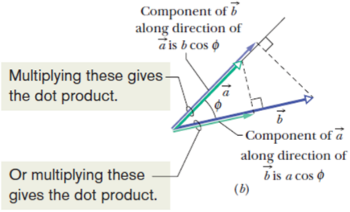

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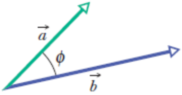
$$\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$$

$$\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0$$

Multiplying Vectors

3. The **vector product** *is the multiplication of two vectors that results in another vector.*



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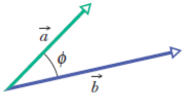

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$$\vec{c} = \vec{a} \times \vec{b}$$
$$c = ab\sin(\phi)$$

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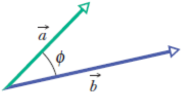


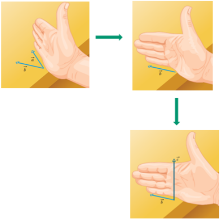
(a)

$$\vec{c} = \vec{a} \times \vec{b}$$
$$c = ab\sin(\phi)$$

Note: $\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$ (anti-commutative)

Multiplying Vectors

The vector product tells us how perpendicular two vectors are. The new vector's direction is given by the **right-hand rule**.



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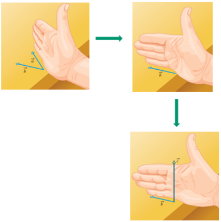

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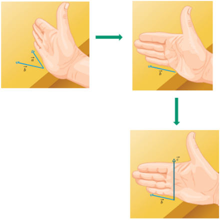

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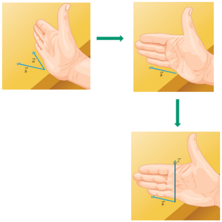

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$$\hat{i} \times \hat{j} = \hat{k} \quad \hat{j} \times \hat{k} = \hat{i} \quad \hat{k} \times \hat{i} = \hat{j}$$

Multiplying Vectors

Lecture Question 2.3

For the two vectors $\vec{A} = 1.1\hat{i} + 2.0\hat{j}$ and $\vec{B} = 1.0\hat{i} - 1.0\hat{j}$, find $\vec{A} \cdot \vec{B}$.

(a) zero

(b) -0.9

(c) $1.1\hat{i} - 2.0\hat{j}$

(d) 3.1

(e) $0.1\hat{i} + 1.0\hat{j}$