# Near Quantum Limited Optical Phase Measurements on a Dark Fringe

**David J. Starling**
*Grad students:* P. Ben Dixon, Nathan S. Williams
*Advisers:* Andrew N. Jordan, John C. Howell

University of Rochester
Department of Physics and Astronomy

# Outline

- Weak Values (WV)
- Homodyne
- WV Phase Measurement and SNR
- Conclusion
- References

Supported by:
Army Research Office Grant No. W911NF-09-0-01417
DARPA Expansion Grant No. N00014-08-1-120

1850 MELIORA
UNIVERSITY of ROCHESTER

# Weak Values

- The result of a pre- and post-selected weak measurement

# Weak Values

- The result of a pre- and post-selected weak measurement
- The weak value is likened to an eigenvalue – however, the weak value can lie outside of the normal eigenvalue range

# First paper - 1988

VOLUME 60, NUMBER 14 PHYSICAL REVIEW LETTERS 4 APRIL 1988

**How the Result of a Measurement of a Component of the Spin of a Spin-$\frac{1}{2}$ Particle Can Turn Out to be 100**

Yakir Aharonov, David Z. Albert, and Lev Vaidman
*Physics Department, University of South Carolina, Columbia, South Carolina 29208, and School of Physics and Astronomy, Tel-Aviv University, Ramat Aviv 69978, Israel*
(Received 30 June 1987)

We have found that the usual measuring procedure for preselected and postselected ensembles of quantum systems gives unusual results. Under some natural conditions of weakness of the measurement, its result consistently defines a new kind of value for a quantum variable, which we call the weak value. A description of the measurement of the weak value of a component of a spin for an ensemble of preselected and postselected spin-$\frac{1}{2}$ particles is presented.

PACS numbers: 03.65.Bz

[2] Y. Aharonov, D. Z. Albert, L. Vaidman, *Phys. Rev. Lett.* **60**, 1351 (1988)

# Weak Values?

- The result of a pre- and post-selected weak measurement
- The weak value is likened to an eigenvalue – however, the weak value can lie outside of the normal eigenvalue range
- Had a rough start – problem of interpretation

# Weak Values?

- The result of a pre- and post-selected weak measurement
- The weak value is likened to an eigenvalue – however, the weak value can lie outside of the normal eigenvalue range
- Had a rough start – problem of interpretation
- Can have an amplification effect, and reduce technical noise [3-5]

# Amplification

## Observation of the Spin Hall Effect of Light via Weak Measurements

**Onur Hosten* and Paul Kwiat**

We have detected a spin-dependent displacement perpendicular to the refractive index gradient for photons passing through an air-glass interface. The effect is the photonic version of the spin Hall effect in electronic systems, indicating the universality of the effect for particles of different nature. Treating the effect as a weak measurement of the spin projection of the photons, we used a preselection and postselection technique on the spin state to enhance the original displacement by nearly four orders of magnitude, attaining sensitivity to displacements of ~1 angstrom. The spin Hall effect can be used for manipulating photonic angular momentum states, and the measurement technique holds promise for precision metrology.

This effect is different from (i) the previously measured (*5*) longitudinal Goos-Hänchen (*6*) and transverse Imbert-Fedorov (*7*, *8*) shifts in total internal reflection, which are described in terms of evanescent wave penetration, and (ii) the recently reported "optical spin Hall effect," which deals with optically generated spin currents of exciton-polaritons in a semiconductor microcavity (*9*). The splitting in the SHEL, implied by angular momentum conservation, takes place as a result of an effective spin-orbit interaction. The same interaction also leads to other effects such as the optical Magnus effect (*10*, *11*), the fine-splitting of the energy levels of an optical resonator (*12*) [in which the interaction resembles

[3] O. Hosten, P. Kwiat, Science **319,** 787 (2008).

# Amplification

**Observation of the Spin Hall Effect**

This effect is different from (i) the previously measured (*5*) longitudinal Goos-Hänchen (*6*)

Selected for a Viewpoint in *Physics*
PRL **102,** 173601 (2009) PHYSICAL REVIEW LETTERS week ending 1 MAY 2009

**Ultrasensitive Beam Deflection Measurement via Interferometric Weak Value Amplification**

P. Ben Dixon, David J. Starling, Andrew N. Jordan, and John C. Howell
*Department of Physics and Astronomy, University of Rochester, Rochester, New York 14627, USA*
(Received 12 January 2009; published 27 April 2009)

We report on the use of an interferometric weak value technique to amplify very small transverse deflections of an optical beam. By entangling the beam's transverse degrees of freedom with the which-path states of a Sagnac interferometer, it is possible to realize an optical amplifier for polarization independent deflections. The theory for the interferometric weak value amplification method is presented along with the experimental results, which are in good agreement. Of particular interest, we measured the angular deflection of a mirror down to 400 ± 200 frad and the linear travel of a piezo actuator down to 14 ± 7 fm.

[3] O. Hosten, P. Kwiat, Science **319,** 787 (2008).
[4] P. B. Dixon *et al.*, *Phys. Rev. Lett.* **102**, 173601 (2009)

# Amplification

**Observation of the Spin Hall Effect**

This effect is different from (i) the previously measured (*5*) longitudinal Goos-Hänchen (*6*)

Selected for a Viewpoint in *Physics*

PRL **102,** 173601 (2009) PHYSICAL REVIEW LETTERS week ending 1 MAY 2009

RAPID COMMUNICATIONS

PHYSICAL REVIEW A **80**, 041803(R) (2009)

**Optimizing the signal-to-noise ratio of a beam-deflection measurement with interferometric weak values**

David J. Starling, P. Ben Dixon, Andrew N. Jordan, and John C. Howell

*Department of Physics and Astronomy, University of Rochester, Rochester, New York 14627, USA*

(Received 29 June 2009; published 8 October 2009)

The amplification obtained using weak values is quantified through a detailed investigation of the signal-to-noise ratio for an optical beam-deflection measurement. We show that for a given deflection, input power and beam radius, the use of interferometric weak values allows one to obtain the optimum signal-to-noise ratio using a coherent beam. This method has the advantage of reduced technical noise and allows for the use of detectors with a low saturation intensity. We report on an experiment which improves the signal-to-noise ratio for a beam-deflection measurement by a factor of 54 when compared to a measurement using the same beam size and a quantum-limited detector.

[3] O. Hosten, P. Kwiat, Science **319,** 787 (2008).
[4] P. B. Dixon *et al.*, *Phys. Rev. Lett.* **102**, 173601 (2009)
[5] D. J. Starling *et al.*, *Phys. Rev. A* **82**, 041803(R) (2009).

# Press

Physics

*Physics* **2**, 32 (2009)

## Viewpoint

## Weak measurements just got stronger

**Sandu Popescu**
*H. H. Wills Physics Laboratory, University of Bristol, Bristol BS8 1TL, UK*

Published April 27, 2009

*In the weird world of quantum mechanics, looking at time flowing backwards allows us to look forward to precision measurements.*

Subject Areas: **Optics, Quantum Mechanics**

**A Viewpoint on:**
**Ultrasensitive Beam Deflection Measurement via Interferometric Weak Value Amplification**
P. Ben Dixon, David J. Starling, Andrew N. Jordan and John C. Howell
*Phys. Rev. Lett.* **102,** 173601 (2009) – Published April 27, 2009

# Press

Physics

*Physics* **2**, 32 (2009)

NATURE|Vol 463|18 February 2010

QUANTUM MEASUREMENT

## A light touch

Aephraim M. Steinberg

**A technique used primarily to study fundamental issues in quantum mechanics has now been shown to have promise as a powerful practical tool for making ultra-precise measurements.**

# Press

Physics

*Physics* **2**, 32 (2009)

NATURE | Vol 463 | 18 February 2010

# BACK FROM THE FUTURE

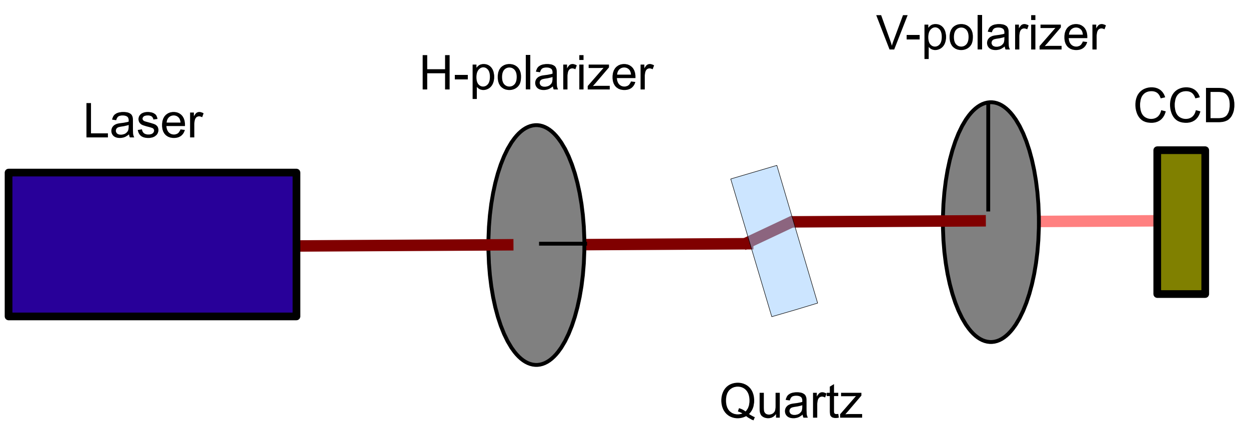
Does the universe have a destiny—
and could the laws of physics be pulling us
inexorably toward our prewritten fate?

# Measurement of Birefringence [1]

- Couple photon polarization to position

[1] N. W. M. Ritchie, J. G. Story, R. G. Hulet, *Phys. Rev. Lett.* **66**, 1107 (1991).

# Measurement of Birefringence [1]



# Measurement of Birefringence [1]

- Couple photon polarization to position
- Pre-select on $|\Psi_i\rangle = |H\rangle \otimes |\psi\rangle = \frac{|F\rangle - |S\rangle}{\sqrt{2}} \otimes |\psi\rangle$

[1] N. W. M. Ritchie, J. G. Story, R. G. Hulet, *Phys. Rev. Lett.* **66**, 1107 (1991).

# Measurement of Birefringence [1]

- Couple photon polarization to position
- Pre-select on $$|\Psi_i\rangle = |H\rangle \otimes |\psi\rangle = \frac{|F\rangle - |S\rangle}{\sqrt{2}} \otimes |\psi\rangle$$
- Post-select on $$|\Psi_f\rangle = |V + \delta\rangle \otimes |\psi\rangle \approx \frac{(1+\delta)|F\rangle + (1-\delta)|S\rangle}{\sqrt{2}} \otimes |\psi\rangle$$

[1] N. W. M. Ritchie, J. G. Story, R. G. Hulet, *Phys. Rev. Lett.* **66**, 1107 (1991).

# Measurement of Birefringence [1]

- Couple photon polarization to position
- Pre-select on $$|\Psi_i\rangle = |H\rangle \otimes |\psi\rangle = \frac{|F\rangle - |S\rangle}{\sqrt{2}} \otimes |\psi\rangle$$
- Post-select on $$|\Psi_f\rangle = |V + \delta\rangle \otimes |\psi\rangle \approx \frac{(1+\delta)|F\rangle + (1-\delta)|S\rangle}{\sqrt{2}} \otimes |\psi\rangle$$
- Between: small polarization-dependent shift of position, called the “measurement”

$$U = e^{-ik_x \mathbf{A} \epsilon_x}$$

$$\mathbf{A} = |F\rangle\langle F| - |S\rangle\langle S|$$

$$\epsilon_x \ll \sigma$$

$$\langle \Psi_f | U | \Psi_i \rangle \propto \int dk_x \, \Phi(k_x) |k_x\rangle e^{-ik_x A_w \epsilon_x}$$

where $|A_w| = \left| \frac{\langle \psi_f | \mathbf{A} | \psi_i \rangle}{\langle \psi_f | \psi_i \rangle} \right| \approx 1/\delta$

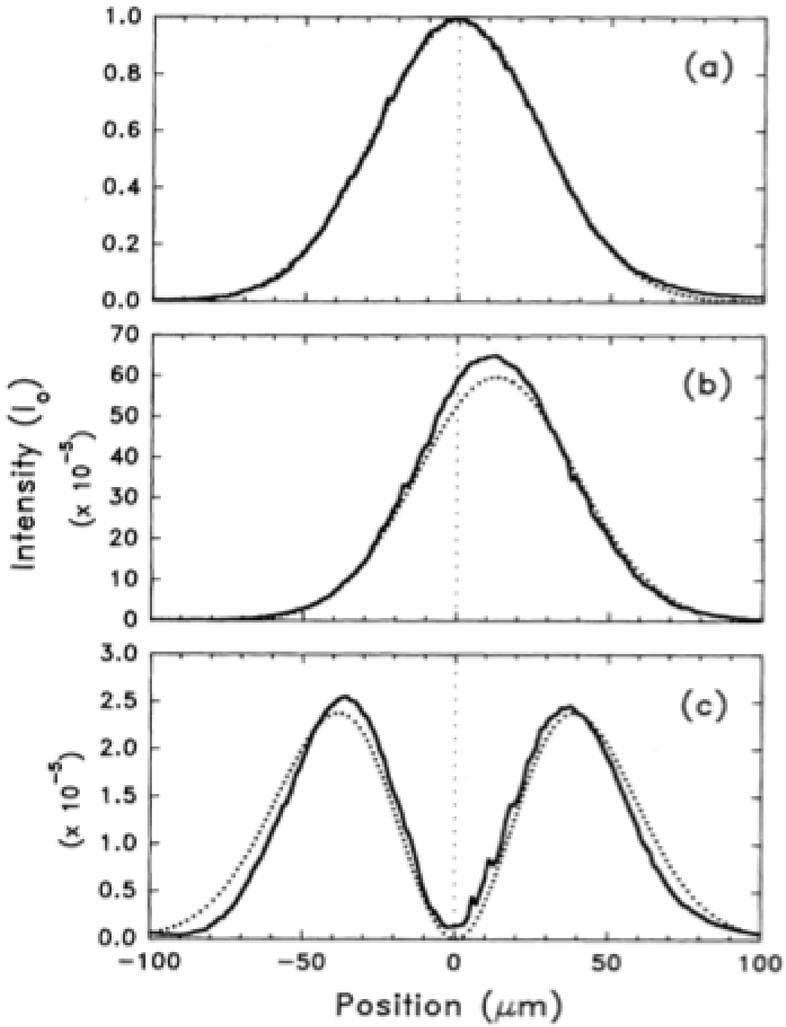
[1] N. W. M. Ritchie, J. G. Story, R. G. Hulet, *Phys. Rev. Lett.* **66**, 1107 (1991).

# Measurement of Birefringence [1]

- Some data from [1]

$$|A_w| \gg 1 \longrightarrow$$

$$\delta = 0 \longrightarrow$$

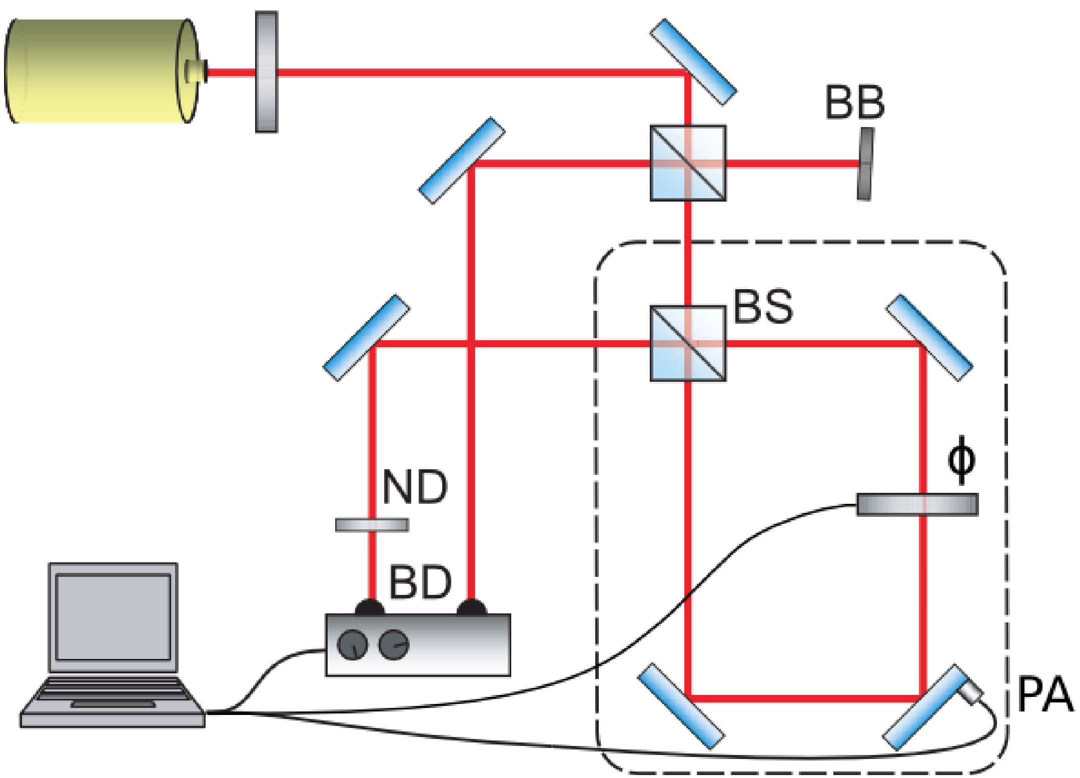




[1] N. W. M. Ritchie, J. G. Story, R. G. Hulet, *Phys. Rev. Lett.* **66**, 1107 (1991).

# Phase Measurement using Homodyne

- A Sagnac interferometer



# Homodyne

- The signal output from the balance detector, when the interferometer is *balanced* is given by

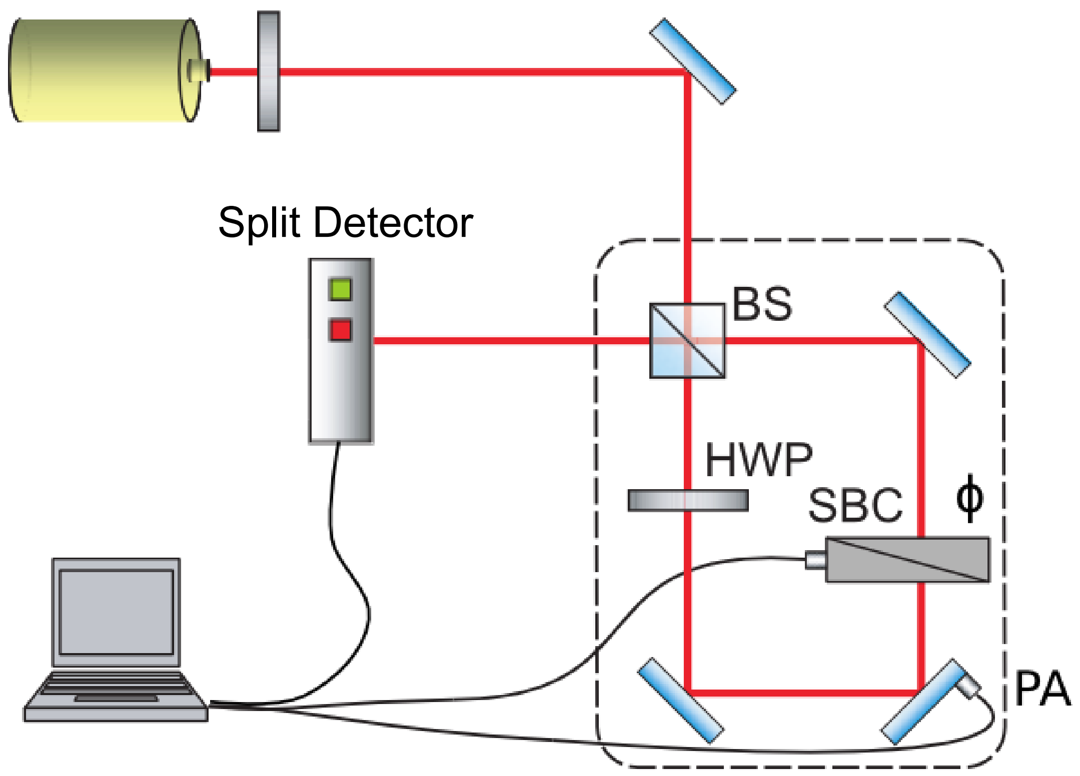
$$\Delta = \sin(\phi) \quad \text{(normalized)}$$

- The signal to noise ratio for a coherent laser source is simply

$$\mathcal{R} = \sqrt{N}\sin(\phi)$$

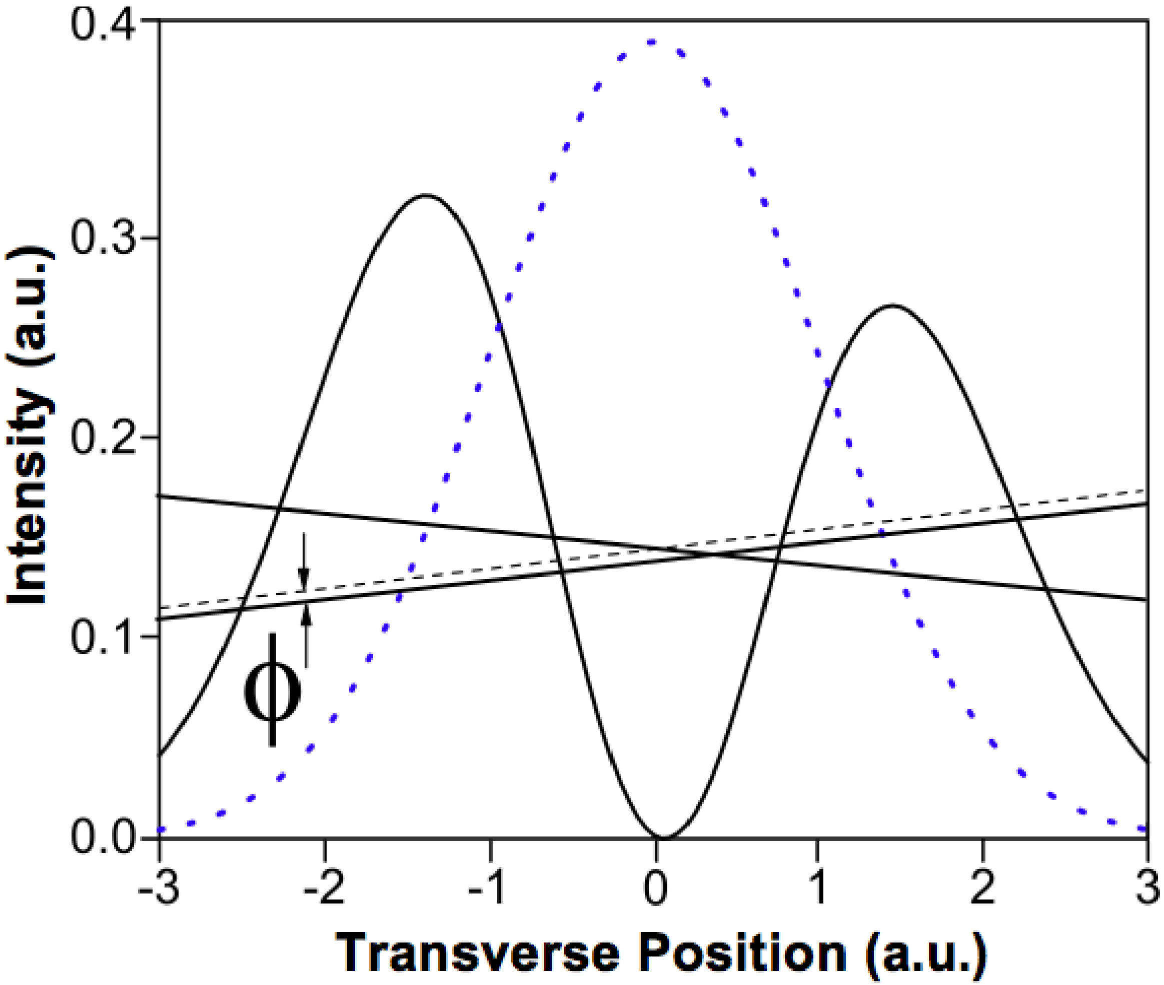
# Phase Measurement using Split Detection [6]

- A similar Sagnac




[6] D. J. Starling *et al.*, *Phys. Rev. A* **82**, 011802(R) (2010).

# Beam Profile



# Weak Value Phase Amplification

- System states: $\mathbf{A} = |\circlearrowright\rangle\langle\circlearrowright| - |\circlearrowleft\rangle\langle\circlearrowleft|$

# Weak Value Phase Amplification

- System states: $\mathbf{A} = |\circlearrowright\rangle\langle\circlearrowright| - |\circlearrowleft\rangle\langle\circlearrowleft|$
- Same weak value: $A_w = \frac{\langle\psi_f|\mathbf{A}|\psi_i\rangle}{\langle\psi_f|\psi_i\rangle}$

# Weak Value Phase Amplification

- System states: $\mathbf{A} = |\circlearrowright\rangle\langle\circlearrowright| - |\circlearrowleft\rangle\langle\circlearrowleft|$
- Same weak value: $A_w = \frac{\langle\psi_f|\mathbf{A}|\psi_i\rangle}{\langle\psi_f|\psi_i\rangle}$
- $k\sigma \ll \phi \ll 1$ (deflection) $\longrightarrow$ $\phi \ll k\sigma \ll 1$

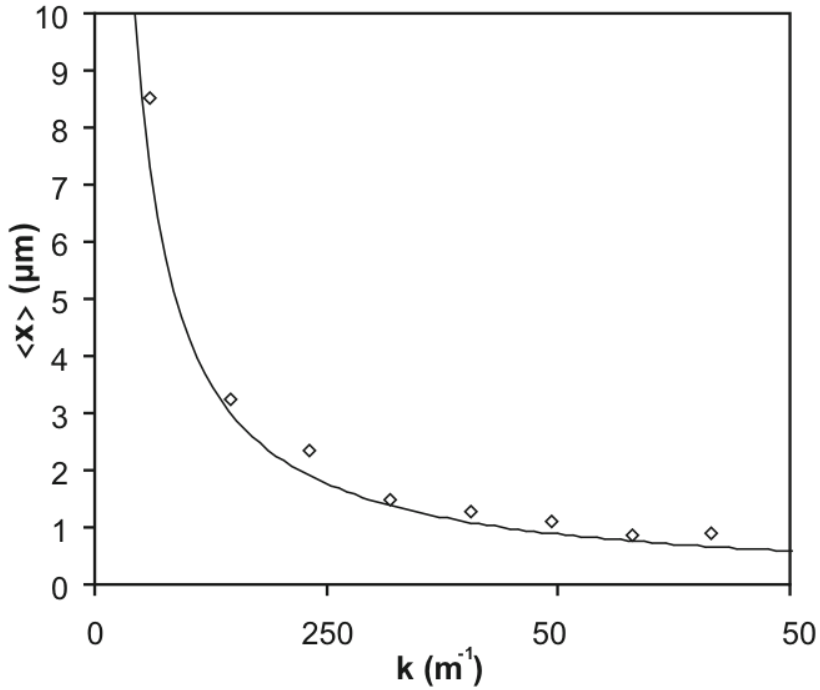
# Weak Value Phase Amplification

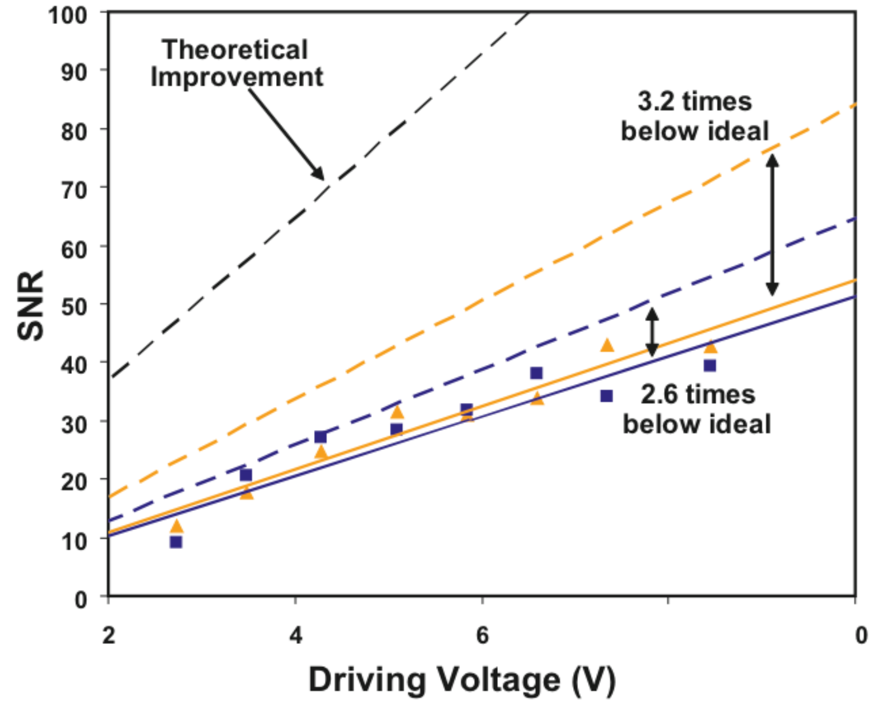
- System states: $\mathbf{A} = |\circlearrowright\rangle\langle\circlearrowright| - |\circlearrowleft\rangle\langle\circlearrowleft|$
- Same weak value: $A_w = \dfrac{\langle\psi_f|\mathbf{A}|\psi_i\rangle}{\langle\psi_f|\psi_i\rangle}$
- $k\sigma \ll \phi \ll 1$ (deflection) $\longrightarrow$ $\phi \ll k\sigma \ll 1$
- Amplification of phase by transverse kick $k$
  - $\langle x\rangle = -2\,\mathrm{Im}[A_w^{-1}]/k \approx -\phi/k$

# Weak Value Phase Amplification

- System states: $\mathbf{A} = |\circlearrowright\rangle\langle\circlearrowright| - |\circlearrowleft\rangle\langle\circlearrowleft|$
- Same weak value: $A_w = \dfrac{\langle\psi_f|\mathbf{A}|\psi_i\rangle}{\langle\psi_f|\psi_i\rangle}$
- $k\sigma \ll \phi \ll 1$ (deflection) $\longrightarrow$ $\phi \ll k\sigma \ll 1$
- Amplification of phase by transverse kick $k$
  - $\langle x\rangle = -2\,\mathrm{Im}[A_w^{-1}]/k \approx -\phi/k$
- SNR: $\mathcal{R} = \sqrt{\dfrac{2N}{\pi}}\sin(\phi)$

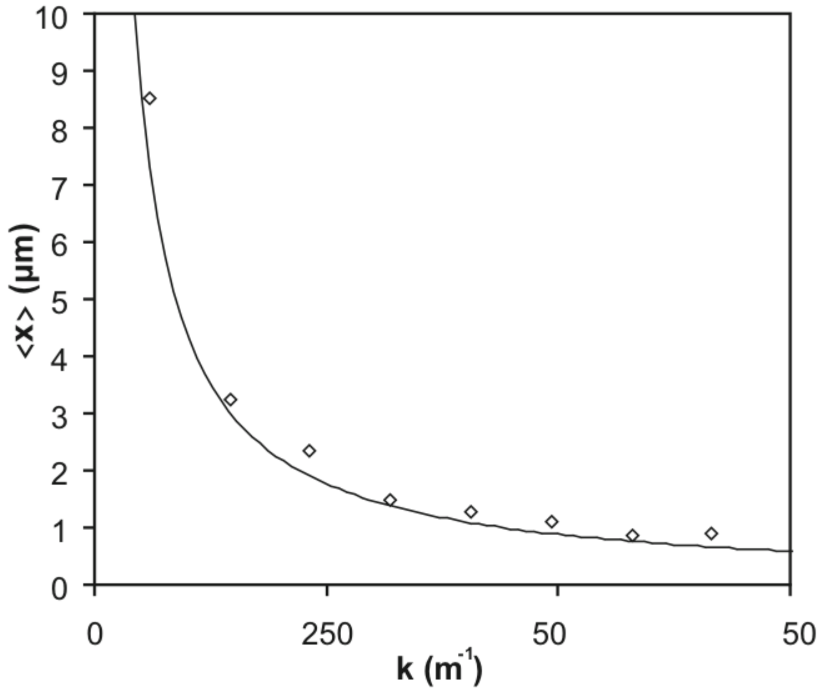
# Results

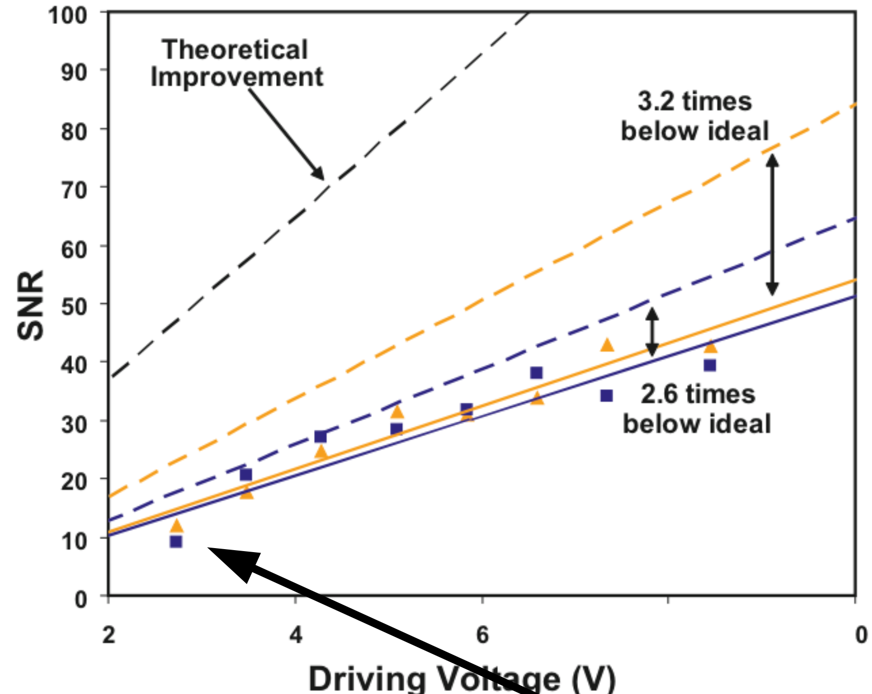



- Clear Inverse dependence on *k*
- Similar sensitivities using both methods
- Much less incident power on detector

# Results




- Clear Inverse dependence on *k*
- Similar sensitivities using both methods
- Much less incident power on detector

# Conclusion

- Split-detection with a coherent split-mode beam has nearly the same quantum limit for phase sensitivity as balanced homodyne
- When detector saturation is the limiting factor, this technique allows for an improvement by a factor of $1/\sqrt{P_{ps}}$

# References


[1] N. W. M. Ritchie, J. G. Story, R. G. Hulet, *Phys. Rev. Lett.* **66**, 1107 (1991).

[2] Y. Aharonov, D. Z. Albert, L. Vaidman, *Phys. Rev. Lett.* **60**, 1351 (1988)

[3] O. Hosten, P. Kwiat, Science **319,** 787 (2008).

[4] P. B. Dixon, D. J. Starling, A. N. Jordan, J. C. Howell, *Phys. Rev. Lett.* **102***,* 173601 (2009)

[5] D. J. Starling, P. B. Dixon, A. N. Jordan, J. C. Howell, *Phys. Rev. A* **82**, 041803(R) (2009).

[6] D. J. Starling, P. B. Dixon, Nathan S. Williams, A. N. Jordan, J. C. Howell, *Phys. Rev. A* **82**, 011802(R) (2010).