CONTROL PROBLEM: You should do this problem independently without consulting other students.

36. Consider a Riemannian metric in local coordinates
\[ ds^2 = a(x, y)dx^2 + 2b(x, y)dxdy + c(x, y)dy^2. \]
Interpret the following conditions in terms of the coefficients of the metric:

1. The coordinate curves \( x = \text{const} \) and \( y = \text{const} \) are orthogonal;
2. The coordinate curves \( x = \text{const} \) and \( y = \text{const} \) form the angle \( \pi/4 \) at each point;
3. The area determined by the metric coincides with the usual area \( dxdy \).

REGULAR PROBLEMS:

NOTICE: Problem N37 has been replaced.

37. Prove the following formula for the hyperbolic distance between two points \( z_1 \) and \( z_2 \) in the upper half-plane
\[ d(z_1, z_2) = \ln \left( \frac{|z_1 - \bar{z}_2| + |z_1 - z_2|}{|z_1 - \bar{z}_2| - |z_1 - z_2|} \right). \]

38. Given a smooth function \( F \) on a surface \( S \) with Riemannian metric one defines (Riemannian) gradient \( \nabla F \) of \( F \) as follows:
At any non-critical point \( x \) there is unique direction of fastest increase of \( F \) i.e a tangent vector \( v \in T_x S \) such that the derivative \( D_v F \) of \( F \) along \( v \), i.e. along any parametrized curve tangent to \( v \), is maximal among all derivatives along tangent vectors of unit length.
(i) Prove this.
Then
\[ \nabla F(x) = \begin{cases} 
D_vF \cdot v, & \text{if } x \text{ is non-critical} \\
0, & \text{if } x \text{ is critical.}
\end{cases} \]

(ii) Prove that \( \nabla F \) is a smooth vector field orthogonal to the level curves of the function \( F \) at all non-critical points.

*Hint: Use local coordinates.*