32. Prove the generalization of Lebesgue Density Point Theorem for arbitrary finite Borel measure on an interval.

   *Hint:* The leading special case in a non-atomic measure with full support.

33. Suppose that for a measurable set $A$ and $x \in A$ one can find a sequence $h_n(x) \to 0$ such that $\liminf_{n \to \infty} \frac{h_{n+1}(x)}{h_n(x)} > 0$ and

\[
\lim_{n \to \infty} \frac{\lambda(A \cap [x - h_n(x), x + h_n(x)])}{2h_n(x)} = 1.
\]

Prove that

\[
\lim_{h \to 0} \frac{\lambda(A \cap [x - h, x + h])}{2h} = 1.
\]

Here $\lambda$ is Lebesgue measure.

34. Suppose that $\mu$ is a Borel measure on $[0, 1]$ such that for a certain constant $C$ and every interval $I$, $\mu(I) < C\lambda(I)$. Prove that there exists a bounded Lebesgue measurable function $\rho$ such that for every $f \in L^1([0, 1], \lambda)$,

\[
\int f d\mu = \int \rho f d\lambda.
\]

35. Prove that every complex-valued function $f$ on the real line which satisfies Lipschitz condition (i.e $|f(x) - f(y)| \leq C|x - y|$ for some $C > 0$ and all $x, y \in \mathbb{R}$) is almost everywhere differentiable.

**An extra credit problem**

6E. Formulate and prove a counterpart for the Lebesgue Density Point Theorem for the Lebesgue measure in $\mathbb{R}^2$.

   *Hint:* You may use the standard Peano curve and an argument similar to Problem 33.