In all problems the measure space is assumed to be finite.

28.

29. Let $1 \leq p \leq q \leq \infty$. Prove that the unit ball in $L^q(X, \mu)$ (i.e., the set of functions whose $L^q$ norm is $\leq 1$) is a closed subset of $L^p(X, \mu)$.

30. Construct an orthonormal basis in $L^2([0,1],\lambda)$ whose elements are simple functions.

31. An element $x$ of a Banach space $B$ is called an extreme point of the set $S$ if from $x = \alpha x_1 + (1 - \alpha)x_2$, $0 < \alpha < 1$, $x_1, x_2 \in S$ it follows that $x = x_1 = x_2$.

Prove that for non-atomic $\mu$ the (closed) unit ball in $L^1(X, \mu)$ does not have any extreme points.

An extra credit problem

5E. Prove that for $p > 1$ any function of $L^p$ norm one is an extreme point of the unit ball in $L^p(X, \mu)$. 