17. Consider a measurable set \( A \subset [0,1] \) of positive Lebesgue measure. Let \( \lambda_A \) be the normalized restriction of Lebesgue measure to \( A \): for a measurable \( B \subset A \), \( \lambda_A(B) = \frac{\lambda(B)}{\lambda(A)} \). Prove that \( (A, \lambda_A) \) is a Lebesgue space. 

**Hint:** One of possible approaches is to represent \( A \) as an \( F_\sigma \) set \( \mod 0 \).

18. Consider a \( \sigma \)-algebra of subsets of \( X \) with non-atomic finite measure \( \mu \). Show that for every \( t, 0 < t < \mu(X) \) there exist uncountably many measurable sets of measure \( t \) pairwise different \( \mod 0 \), i.e. for any two sets \( A, B \) from the collection \( \mu(A \Delta B) > 0 \).

**Hint:** First construct one such set.

19. Let for \( 0 < p < 1 \), \( \beta_p \) be the measure on \( \{0,1\} \) such that \( \beta_p(\{0\}) = p \) and \( \beta_p(\{0\}) = 1 - p \). Consider the countable product of measures \( \mu_{p_n} \) on the space \( \Omega_2 \). Find a necessary and sufficient condition on the sequence \( p_n \) for the space \( \Omega_2 \) with this measure to be a Lebesgue space.

20. Consider a measurable function \( f : [0,1] \to \mathbb{R} \) and let \( \mathcal{B} \) be the \( \sigma \)-algebra of the sets of the form \( f^{-1}(A) \) where \( A \) is a Lebesgue measurable set on the line. Find necessary and sufficient condition for this algebra to be isomorphic \( \mod 0 \) to Lebesgue \( \sigma \)-algebra.

**An extra credit problem**

4E. Consider a Borel non-atomic measure \( \mu \) on the open unit square \( I^2 \) such that any open set has positive measure. Prove that there exists a homeomorphism \( I^2 \to I^2 \) such that for any Borel set \( A \), \( \mu(A) = \lambda(h(A)) \), where \( \lambda \) is Lebesgue measure.

**Hint:** Try to use an inductive procedure to adjust the measure on finer and finer grids.