15. Greek method for finding square roots. Let $a > 1$ be a rational number. Define two sequences $a_n$ and $b_n$ recursively:

$$a_1 = a, \quad b_1 = 1, \quad a_{n+1} = \frac{a_n + b_n}{2}, \quad b_{n+1} = \frac{2a_nb_n}{a_n + b_n}.$$ 

Prove that both sequences are Cauchy sequences, that they are equivalent and that sequences $a_n^2$ and $b_n^2$ converge to $a$.

16. *) Justify the standard rules for taking limits of the sum, product and ratio for sequences of real numbers using the definition of real numbers as equivalence classes of Cauchy sequences of rationals.

Due on Monday March 15.

17. *) Prove that monotone increasing sequence bounded from above is Cauchy.

Due on Monday March 15.