MATH 312H:
FUNDAMENTAL STRUCTURES OF CONTINUOUS MATHEMATICS

SPRING 2004

A.Katok

PROBLEM LIST #2:

Problems on this list are designed for various purposes: Those marked with *) are homework problems; written solutions are due on the date indicated. Unmarked problems usually will be discussed in class; you should give those problems some thought beforehand. Some of those later may be designated as homework. Problems marked **) are more advanced and optional; both solutions and questions in class or by email about those problems are welcome.

8*). Write down a careful proof of the following statement: If $A$ is any uncountable set and $C \subset A$ a finite or countable subset, then $A$ and $A \setminus C$ have the same power.

Due on Monday January 26.

9*). Prove that the set of all finite subsets of a countable set is countable.

Due on Monday February 2.

10. Prove that the set of all finite subsets of a continuum has the power of continuum. Give a geometric interpretation of this fact.

11. Prove that the set of all countable subsets of a continuum has the power of continuum.

$Hint$: Think of the method used to prove countability of rational numbers (The first Cantor diagonal process).

12. Prove that the set of all subsets of a continuum has power greater than continuum.

$Hint$: Think of the method used to prove uncountability of continuum (The second Cantor diagonal process).

13. Prove that the set of all real–valued functions on the real line has the same power as the set of all subsets of real line.

$Hint$: It is enough to prove that each of the two sets has the same power as a subset of the other.

14**). Prove that the set of all continuous functions on the real line has the power of continuum.