Exercise 1: (1) Let $g$ be a reductive Lie algebra. Show that $[g, g]$ is semi-simple.

(2) Let $h$ be an ideal of a Lie algebra $g$. Show that if both $h$ and $g/h$ are semi-simple then $g$ is semi-simple.

Exercise 2: Let $\mathfrak{h} \subset \mathfrak{so}(4, \mathbb{C})$ be the subalgebra consisting of matrices of the form:

$$A = \begin{pmatrix}
0 & a & 0 & 0 \\
-a & 0 & 0 & 0 \\
0 & 0 & 0 & b \\
0 & 0 & -b & 0
\end{pmatrix}$$

Show that $\mathfrak{h}$ is a Cartan subalgebra and find the corresponding root decomposition.