Exercise 1: Let $V$ and $W$ be irreducible representations of a Lie group $G$. Show that $(V \otimes W^*)^G = \{0\}$ if $V$ is non-isomorphic to $W$, and that $(V \otimes V^*)^G$ is canonically isomorphic to $\mathbb{C}$.

Exercise 2: Prove an analog of Theorem 4.60 for complex representations of $\mathfrak{so}(3, \mathbb{R})$.

Exercise 3: Let $g$ be real Lie algebra with a definite positive Killing form $K : g \times g \to \mathbb{R}$ defined by

$$K(X, Y) = \text{tr}(\text{ad}_X \circ \text{ad}_Y), \forall X, Y, \in g.$$ 

Show that $g = \{0\}$. Hint: $g \subset \mathfrak{so}(g)$.

Exercise 4: Show that for $g = \mathfrak{sl}(n, \mathbb{C})$, the Killing form is given by

$$K(X, Y) = 2n \text{ tr}(XY), \forall X, Y, \in g.$$ 

Exercise 5: Let $\mathfrak{b}(n, \mathbb{C}) \subset \mathfrak{gl}(n, \mathbb{C})$ be the subalgebra that consists of all upper triangular matrices. Show that $\mathfrak{b}(n, \mathbb{C})$ is solvable but it’s not nilpotent.