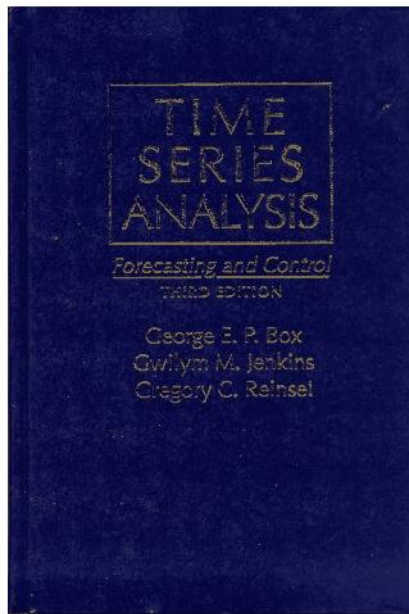


# SARIMA Models



# Outline

## 1 SARIMA

# Seasonal ARMA( $P, Q$ )

Seasonal ARMA( $P, Q$ ) is used when seasonal (hence nonstationary) behavior is present in the time series.

We use the model

$$\Phi_P(B^s)Z_t = \Theta_Q(B^s)a_t$$

where  $s = 12$  if data is in months and  $s = 4$  if data is in quarters, etc.

Seasonal differencing may be in order if the seasonal component follows a random walk, as in

$$Z_t = Z_{t-12} + a_t$$

The seasonal difference of order  $D$  is defined as

$$\nabla_s^D Z_t = (1 - B^s)^D Z_t$$

# SARIMA Model

## Definition (SARIMA Model)

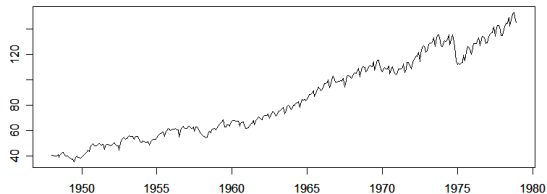
The seasonal autoregressive integrated moving average model of Box and Jenkins (1970) is given by

$$\Phi_P(B^s)\phi(B)\nabla_s^D\nabla^d Z_t = \alpha + \Theta_Q(B^s)\theta(B)a_t$$

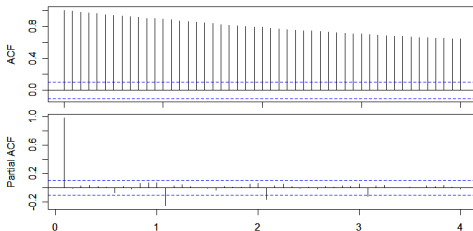
and is denoted as an  $\text{ARIMA}(p, d, q) \times (P, D, Q)_s$ .

# Federal Reserve Board Production Index

```
> prod=ts(scan("mydata/prod.dat"), start=1948, frequency=12)
> ts.plot(prod)
```

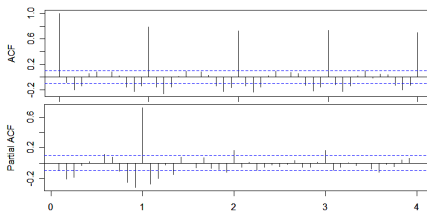


```
> par(mfrow=c(2,1))
> acf(prod, 48)
> pacf(prod, 48)
```

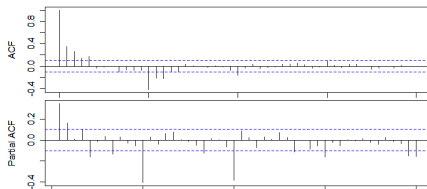


# Federal Reserve Board Production Index

```
par(mfrow=c(2,1)) # (P)ACF of d1 data
acf(diff(prod), 48)
pacf(diff(prod), 48)
```



```
par(mfrow=c(2,1)) # (P)ACF of d1-d12 data
acf(diff(diff(prod),12), 48)
pacf(diff(diff(prod),12), 48)
```



# Federal Reserve Board Production Index

```
> prod.fit3 = arima(prod, order=c(1,1,1),  
+ seasonal=list(order=c(2,1,1), period=12))  
> prod.fit3 # to view the results
```

Call:

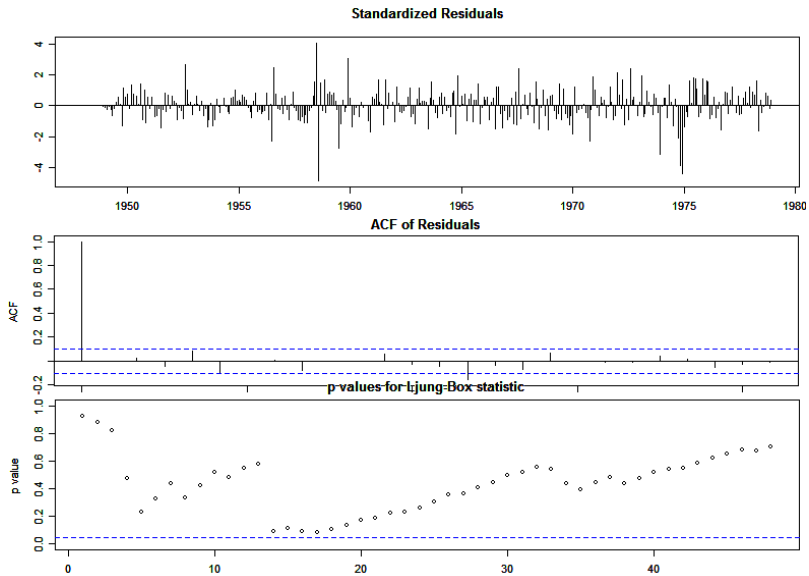
```
arima(x = prod, order = c(1, 1, 1), seasonal = list(order = c(2, 1,
```

Coefficients:

	ar1	ma1	sar1	sar2	sma1
	0.5753	-0.2709	-0.2153	-0.2800	-0.4968
s.e.	0.1120	0.1300	0.0784	0.0619	0.0712

```
sigma^2 estimated as 1.351: log likelihood = -568.22, aic = 1148
```

```
> tsdiag(prod.fit3, gof.lag=48) # diagnostics
```





# Federal Reserve Board Production Index

```

> prod.pr = predict(prod.fit3, n.ahead=12)
> U = prod.pr$pred + 2*prod.pr$se
> L = prod.pr$pred - 2*prod.pr$se
> ts.plot(prod, prod.pr$pred, col=1:2, type="o", ylim=c(105,175), xlab="Year")
> lines(U, col="blue", lty="dashed")
> lines(L, col="blue", lty="dashed")

```

