1) Let $X$ be a space with the trivial topology. Find all the continuous maps $f : X \to \mathbb{R}$. Do the same if $X$ has the discrete topology.

2) Let $F$ be a family of continuous, real-valued functions on a compact, Hausdorff space $X$. Suppose $F$ separate points in $X$. Then every continuous, real-valued function on $X$ can be uniformly approximated by a polynomial in a finite number of functions of $F$.

3) Let $f : E \to \mathbb{R}$. Suppose $E = A \cup B$, $A, B$ measurable sets. Show $f$ is measurable if and only if $f|_A$, $f|_B$ is measurable.

4) Let $\langle f_n \rangle$ be a sequence of non-negative measurable functions that converge to $f$ and
Suppose \( f_u \leq f \) for each \( u \). Then:
\[
\int f = \lim_{u \to \infty} \int f_u.
\]

5) Let \( f \) be a function of two variables \( (x, t) \) defined on the square \( \mathcal{A} = \{(x, t) \mid 0 \leq x \leq 1, 0 \leq t \leq 1\} \) such that \( f(x, t) \) is nonnegative for each \( t \).
   Suppose \( \lim_{t \to 0} f(x, t) = f(x) \) and \( |f(x, t)| \leq g(x) \)
   \( g \) integrable. Then:
   \[
   \lim_{t \to 0} \int f(x, t) \, dt = \int f(x) \, dx.
   \]

6) Let \( g \) be defined by \( g(0) = 0, \quad g(x) = x^2 \sin^2\left(\frac{1}{x}\right) \)
   for \( x \neq 0 \). Is \( f \) of bounded variation on \([-1, 1]\)?

7) Construct an absolutely continuous function, strictly monotone on \([0, 1]\) such that \( g' = 0 \)
on a set of positive measure.

8) Let \( \{f_u\} \) be a sequence of functions in \( L^p \),
   \( 1 < p < \infty \), which converges almost everywhere to a function \( f \) in \( L^p \). Suppose \( f \) is s. t. \( \|f\|_p \leq M + u \)
   Then:
   \[
   \int fg = \lim_{u \to \infty} \int f_u g, \quad \forall g \in L^q.
   \]