INSTRUCTIONS

- There are 3 problems on this exam for a total of 100 points.
- **PLEASE, WRITE YOUR SOLUTIONS CLEARLY AND COMPLETELY. ANSWERS WITHOUT PROOFS WILL BE GIVEN NO CREDIT.**
- Unless otherwise stated, you CAN use without proofs the results discussed in class or in the textbook.
- You **may not** use CALCULATORS, BOOKS, or PERSONAL NOTES.
- **Do not** write on the line marked SCORE at the bottom of each page.
- Cellular phones must be turned off at the beginning of the exam.
1. (30 points) Use the definition of limit to PROVE that:

\[ \lim_{n \to \infty} \frac{n}{n^2 + 1} = 0. \]
2. (30 points) Find
\[ \lim_{n \to \infty} (\sqrt{n} + 2 - \sqrt{n}). \]

PROVE that your answer is correct.
3. (40 points) Let $a$ be an irrational number. SHOW using the denseness property of \( \mathbb{Q} \) in \( \mathbb{R} \), that there exists a sequence of rational numbers \((s_n)\) such that

$$\lim_{n \to \infty} s_n = a.$$ 

**Hint:** consider the two numbers $a - \frac{1}{n}$, $a - \frac{1}{2n}$ for each $n \in \mathbb{N}$. 

SCORE: ___________
4. (10 points) EXTRA CREDIT: Let $S$ be a non-empty bounded subset of $\mathbb{R}$. Let $T = \{-s, s \in S\}$. Show that $\inf(T) = -\sup(S)$. 

SCORE: ___________